NORDIC VOLCANOLOGICAL INSTITUTE 8302
UNIVERSITY OF ICELAND

ACCURACY OF TILT OBSERVATIONS THROUGH OPTICAL LEVELING

by

Eysteinn Tryggvason

Reykjavík
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INTRODUCTION

Detection and observation of tectonic processes can be obtained through many different techniques. Most geological observations which are related to tectonics will show past tectonic processes, but observation of present tectonic processes include seismological observations and various repeated geodetic measurements.

This report will discuss repeated precision levelings as a tool to observe tilting of the ground, with emphasis on the significance of these observations to determine tectonic processes. The levelings used for this study were performed for the purpose of determining the rate of ground deformation within the rift zone of Iceland (Tryggvason, 1974), and the technique of measurement and analysis is basically as described by Bomford (1977) for precision leveling.

The term "tectonic processes" may need an explanation. This has to do with movements or deformation of large blocks of near-surface formations, which are driven by less known dynamic processes at depth.

The word "tilt" as used here is equivalent to "tilting of the ground surface".

The repeated levelings will detect vertical displacements of one or more points on the earth's surface relative to other point or points. The observed relative vertical displacements may be of tectonic origin or not. A proof for tectonic origin is:

> Similar ground tilt, or tilt variation over an extended area.
>
> Similar rate of tilt over an extended time span.
>
> Similar observed tilt as inferred from geologic observations.

The variation of observed tilt in space and/or time needs thus be obtained before a judgement can be made on the origin or cause of the observed relative vertical displacements, or tilt. Measurements which are sufficiently extensive to determine positively if tectonic strain occurs, can also be used to determine how representative each single observation is with respect to the tectonic strain.

If tectonic tilt is prooven, a single tilt observation may deviate greatly from the average (regional) tectonic tilt. There may be many different reasons for this, such as the following.

Observational errors.

Relative movements of the reference bench marks with respect to the basement rock.

Geological inhomogeneity of the basement rock.

Thermal strain at the earth's surface.

Temporal variations of the tectonic processes.

The observational error in optical levelings will not be evaluated here, but many authors have discussed this subject thoroughly as Bomford (1977), Mark et al. (1981) and Strange (1981).

The geological inhomogeneity includes surface irregularities, such as topography, which will modify the regional strain in the vicinity of fault scarps, steep slopes and other topographic features.

Tilt observations by repeated levelings do not separate the different causes of bench mark displacements. However, extensive repeated levelings may be treated statistically to obtain a reasonable estimate of the various components. Multiple repetition of precision levelings along a 3.3 km line and on a dozen circular array tilt stations in the North Iceland rift zone in 1966 to 1982 (Tryggvason, 1974, 1978, 1979, 1981), offer an excellent opportunity to study the various types of errors which affect these measurements. These measurements were made within an area which experienced great tectonic deformation in 1975 to 1982, and the observations include a great range of tectonic strain.

THE LINEAR REYKJAHEIDI LEVELING PROFILE

A linear profile of precision leveling will normally follow a route of easy access, and the permanent bench marks are placed at locations of firm foundation. These requirements will normally mean, that the bench marks will not lie on straight line, but a compromise must be made, so the near linear profile always follows a path of relatively easy access, and never passes too long stretches of unacceptable bench mark foundations.

The Reykjaheidi profile, North Iceland, is a good example of a "linear" leveling profile (Fig. 1). This profile is about 3.3 km long and consists of 30 permanent bench marks (Tryggvason, 1974). It follows a jeep track across a lava field, covered by a layer of soil, but the lava penetrates the soil at numerous locations. The general direction of the profile is W 7.5 N, measured from the lowest numbered marker (301), and all the bench marks lie within 200 m from a straight line connecting the end markers of the profile.

Repeated levelings of a linear precision leveling profile (or linear tilt profile) will normally detect one component of tilt only, the component which lies parallel to the general direction of the profile. Only very large tilt perpendicular to the profile direction will be detected. However, actual ground tilt component perpendicular to the linear profile will appear in the results of repeated levelings as a source of error, in addition to other error sources.

Two levelings of the profile are needed to obtain information on relative displacements of the bench marks. Tilt of the ground is computed by applying the least squares method to the vertical displacement as linear function of the distance along the profile.

The standard error of observed tilt along linear profiles of numerous bench marks is computed as the standard error of the slope of the "best fitted line".

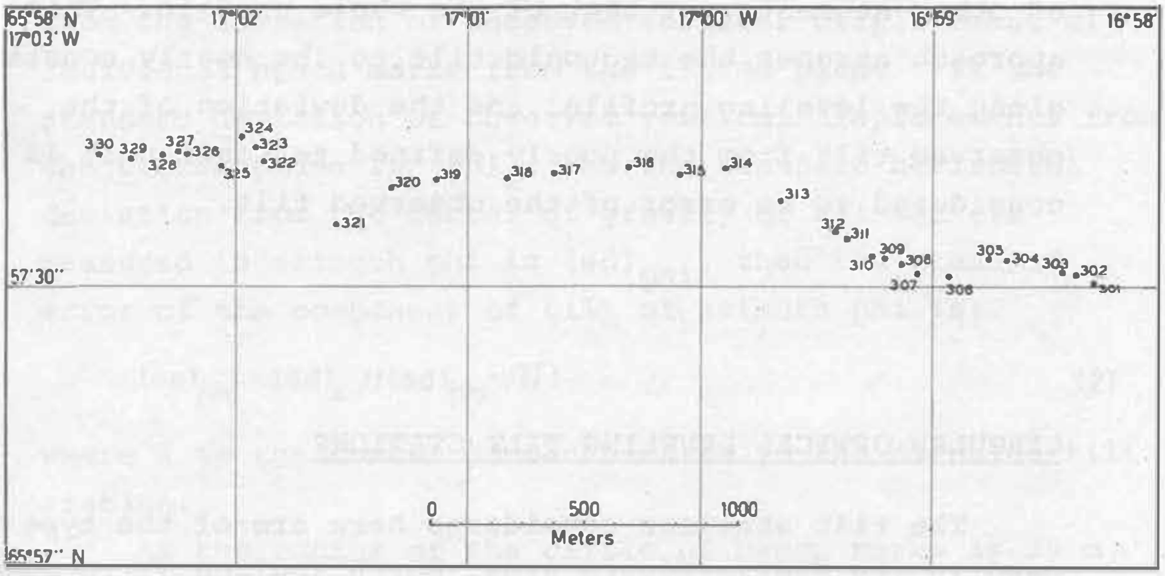

Fig. 1. Map of the Reykjaheidi linear profile of precision leveling. The active zone of rifting during the 1975-(1982) rifting episode lies about 5 km east of the east end of the leveling profile and strikes NNE. The rifting episode has caused uplift and E-W contraction of the flanks of the zone of active rifting.

Thus if the standard deviation of vertical displacements from the best fitted linear tilt is $(sd)_z$ and $(sd)_x$ is the standard deviation of bench marks from the center of gravity of all bench marks, measured along the general direction of the profile, then the standard error of tilt $(s.e.)_t$ is given as:

$$(s.e.)_t = (sd)_z / ((sd)_x \cdot \sqrt{N}) \qquad (1)$$

where N is the number of bench marks used to calculate the tilt.

If the distribution of deviations of observed vertical displacements, from that predicted from the best fitted linear tilt, follows approximately the normal error curve, then tilt errors at any selected confidence level can be obtained from the above standard error by multiplying it by the appropriate constant.

Another approach to estimate the error of observed tilt along a linear profile of leveling is to compare the tilt as computed from segments of the leveling profile, to that of other segments, or that of the whole profile. This approach assumes the tectonic tilt to be nearly constant along the leveling profile, and the deviation of the observed tilt from the poorly defined tectonic tilt is considered to be error of the observed tilt.

CIRCULAR OPTICAL LEVELING TILT STATIONS

The tilt stations considered here are of the type used in the Krafla-Mývatn area, North Iceland (Tryggvason, 1978, 1979). They consist of 5 or 6 permanent bench marks along the periphery of a circle of 25 m radius on a reasonably level ground. Observations are made by placing a tripod with optical level at the center of the circle and an invar leveling rod is carried from one bench mark to another around the circle. The rod is placed twice on each marker, and two readings are taken each time. Thus four readings determine the relative height of each bench mark.

Each observation determines the relative height of each bench mark, and the reference elevation is the average elevation of all bench marks in the circular tilt station. The difference of relative elevation of bench marks between observations is used to determine the ground tilt. This is done by finding by the least squares method, the best linear fit between vertical displacements and horizontal coordinates of the bench marks. For simplicity of calculations, the horizontal coordinates (x and y) are measured from the center of gravity of all bench marks, and the sum of all vertical bench mark displacements (z) is zero because the reference elevation is always the average elevation of all bench marks. By using meter for the x and y coordinates, and micrometer for the z coordinate (relative vertical displacement), the tilt of the ground will be in microradians.

A measure of the apparent tilt error can be obtained from the deviation of observed vertical displacement of individual bench marks from the tilted plane. If the standard deviation of observed vertical displacements from the tilted plane is $(sd)_z$, and the standard horizontal deviation from the center of gravity of all markers measured in azimuth phi is $(sd)_{phi}$, then the standard error of the component of tilt of azimuth phi is:

$$(se)_{phi} = (sd)_z / ((sd)_{phi} \cdot \sqrt{N}) \qquad (2)$$

where N is the number of bench marks in the circular tilt station.

As the radius of the circle of bench marks is 25 m, and the markers are distributed roughly evenly along the periphery of the circle, the value of $(sd)_{phi}$ does not vary greatly with azimuth. For exactly even distance between adjacent markers, the value of $(sd)_{phi}$ will be $25/\sqrt{2}$ or about 17.68 m. This value will be used in the following treatment, although field condition at individual tilt stations require some deviation from the even distribution of markers, changing the true value of $(sd)_{phi}$ to lie between 14 and 20 m. We will, therefore, consider the standard error of tilt at the circular tilt station with 5 bench marks, and 25 m radius as:

$$(s.e.)_t = (sd)_z / 39.5 \qquad (3)$$

where $(s.e.)_t$ is the standard error of tilt in microradians, assumed to be equal along all azimuths, and $(sd)_z$ is measured in micrometers.

The tilt error evaluated above can be caused by erratic movements of the bench marks, irregular surface strain, and observational errors.

ANALYSIS OF TILT ERROR OF THE REYKJAHEIDI PROFILE

The six separate precision levelings of the 3.3 km Reykjaheidi leveling profile in 1966, 1968, 1970, 1972, 1976, and 1980 (Tryggvason, 1974, 1981), allow analysis of tilt errors over various periods of time with greatly variant observed tilt. Further, the large number (30) of permanent bench marks allows the determination of tilt error over variable length of segments of the leveling profile. This makes it possible to estimate the tilt error as a function of the number (N) of bench marks, the length (L) of the profile, the time (T) between levelings, and the observed ground tilt (Ti).

Variation of tilt error with profile length and number of bench marks

The standard error of tilt was calculated for all segments of the Reykjaheidi leveling profile consisting of 4, 6, or 8 adjacent bench marks for the three 2 year periods 1966-1968, 1968-1970, and 1970-1972. The length of the 4-marker segments ranged from 133 to 604 meter, measured along the general direction of the profile. The length of the 6-marker segments ranged from 273 to 944 meter, and that of the 8-marker segments from 429 to 1332 meter. These profile segments allow an estimate of the relation between standard error of tilt, and profile length for 4-marker profiles, 6-marker profiles, and 8-marker profiles. By grouping the profile segments by length into 3 to 5 groups and accepting the average standard error of each group as the most probable standard error of profile of length equal the average length of profile segment in that group, the most probable relation of tilt error and profile length can be obtained.

It is here assumed, that the relation of tilt error and profile length is of the form:

$$\text{s.e.} = A \cdot L^{B} \qquad (4)$$

where s.e. is the standard error of observed tilt, and A and B are constants to be determined.

By applying the least squares method on the grouped data, the most probable values of the constants A and B were found to be:

For 4-marker segments: $A = 0.148$, $B = -0.62$
For 6-marker segments: $A = 0.143$, $B = -0.62$
For 8-marker segments: $A = 0.137$, $B = -0.53$

where s.e. and A are in microradians, and L in km.

The average value of the constant B is very close to -0.6, which is accepted in the following treatment as valid for the linear Reykjaheidi leveling profile.

With the constant B fixed as -0.6, the constant A above can be recalculated, and the most probable values are found to vary with the number of bench marks as follows:

For 4-marker profiles: $A = 0.154$
For 6-marker profiles: $A = 0.145$
For 8-marker profiles: $A = 0.127$
For 30-marker profiles: $A = 0.090$

This constant A is the most probable standard error of tilt in microradians on a one km linear profile. It is apparently a function of N, the number of bench marks in the profile. We will assume that this function is of the form:

$$A = A' N^{B'} \qquad (5)$$

where A′ and B′ are new constants to be determined.

The least squares method, applied to the above data, gives the most probable values as:

$A' = 0.227$ (microradians)
$B' = -0.27$

By combining the effect of profile length and bench mark number on the standard error of tilt over a two year period on the Reykjaheidi leveling profile in the period 1966 to 1972, we arrive at the following equation:

$$s.e. = 0.227 \cdot N^{-0.27} \cdot L^{-0.6} \qquad (6)$$

where s.e. is the standard error of tilt in microradians, and L is the length of the linear leveling profile in km.

The effect of ground tilt on the error of observed tilt

The levelings of the Reykjaheidi profile of precision leveling in 1976 and 1980 show ground tilt which greatly exceeds that observed before 1972 (Tryggvason, 1981). The average tilt component along the profile between the levelings of 1972 and 1976 is calculated as 3.006 (s.e. = 0.169) microradians towards west, and between 1976 and 1980 the calculated tilt is 9.943 (s.e. = 0.534) microradians towards west. The two previous 4-year intervals between levelings (1966-1970and 1968-1972) showed tilt of 1.363 (s.e. = 0.067) and 0.698 (s.e. = 0.039) microradians towards east respectively.

The calculated standard error of tilt as computed from the whole profile (30 bench marks, 3.273 km length) is clearly affected by the observed tilt. It is also clear, that the calculated standard error of tilt is approximately proportional to the observed tilt.

We will assume that the calculated standard error of tilt over a period of 4 years consists of two components, one component which is proportional to the ground tilt, and another component which is independent of tilt. The total calculated standard error of tilt is then expected to be equal the square root of the sum of the two components squared:

$$s.e. = \sqrt{AA^2 + BB^2 \cdot Ti^2} \qquad (7)$$

where AA and BB are constants to be determined and Ti is the observed tilt.

The four 4-year periods allow us to calculate the constants AA and BB by the least squares method for the whole profile, or any segments of the profile.

For the whole profile of 30 bench marks and 3.273 km length, the most probable values of the constants were found to be:

AA = 0.024 mu.rad.
BB = 0.054

We can assume, that the constants AA and BB vary with L (profile length) and N (number of bench marks) according to equation (6). This questionable assumption makes it possible to reduce the numerical value of these constants to unit length (1 km), "unit" bench mark number profile, to obtain new constants A′A′, and B′B′ as follows:

$$A'A' = AA\ f(N,L) \qquad B'B' = BB\ f(N,L) \tag{8}$$

where

$$f(N,L) = N^{0.27} \cdot L^{0.6} \tag{9}$$

The new standardized constants A′A′ and B′B′ for the whole profile are:

A′A′ = 0.122 mu.rad.
B′B′ = 0.276

Similar treatment for segments of 4, 6, and 8 adjacent bench marks gave the average values of the standardized constants A′A′ and B′B′ as 0.378 mu.rad. and 0.246 respectively. A great scatter of the individually calculated standard errors makes these values, especially that of A′A′, quite uncertain. In a previous chapter the average standard error of unit length "unit" number profile was found to be 0.227 mu.rad., if levelings were made two years apart and average tilt was about 0.5 mu.rad. This value corresponds approximately to our constant A′A′. Thus we have 3 determinations of A′A′, 0.122, 0.378, and 0.227, and the average of these three values is 0.24 mu.rad. The two determinations of B′B′ (0.276 and 0.246) give an average value of 0.26. These will be accepted here as the most probable values of A′A′ and B′B′ for the Reykjaheidi linear profile of precision leveling.

The effect of time between levelings on the tilt error

The levelings of 1966, 1968, 1970, and 1972 allow us to compare the calculated standard error of tilt for the whole profile, and segments of it, for 2, 4, and 6 years between levelings.

The average standardized standard error as calculated from all 4-, 6-, and 8-bench mark segments, and for the whole profile gave:

$(s.e.)' = 0.227$ for 2-year periods
$(s.e.)' = 0.277$ for 4-year periods
$(s.e.)' = 0.333$ for 6-year period

where $(s.e.)'$ is the standard error of tilt reduced to unit (1 km) length and "unit" number of bench marks according to equation (6). These numbers indicate a linear relation between standard error of tilt and time between levelings.

In the previous chapter we found that there exists a near linear relation between standard error of tilt and observed tilt. As the average tilt for the 2-, 4-, and 6-year periods was 0.516, 1.031, and 1.547 mu.rad., respectively, the above increase in standard error of tilt with increased time between levelings, can as well be interpreted as caused by the ground tilt itself. Therefore, the present data does not allow us to determine the effect of time between levelings on the standard error of observed tilt. This effect appears to be small if it exists at all, as the previously found relation between tilt error and tilt will fully account for the increase in tilt error with time between levelings in our data.

The final equation giving the observed average standard error of tilt, as function of length (L) of the profile in km, the number (N) of permanent bench marks, and the ground tilt (Ti) in microradians along the profile then becomes (Fig. 2):

$$s.e. = N^{-0.27} \cdot L^{-0.6} \sqrt{0.24^2 + (0.26\ Ti)^2} \qquad (10)$$

This equation cannot be expected to be valid for tilt profiles, other than the Reykjaheidi linear profile of precision leveling, but similar relation should be valid for other tilt profiles or arrays.

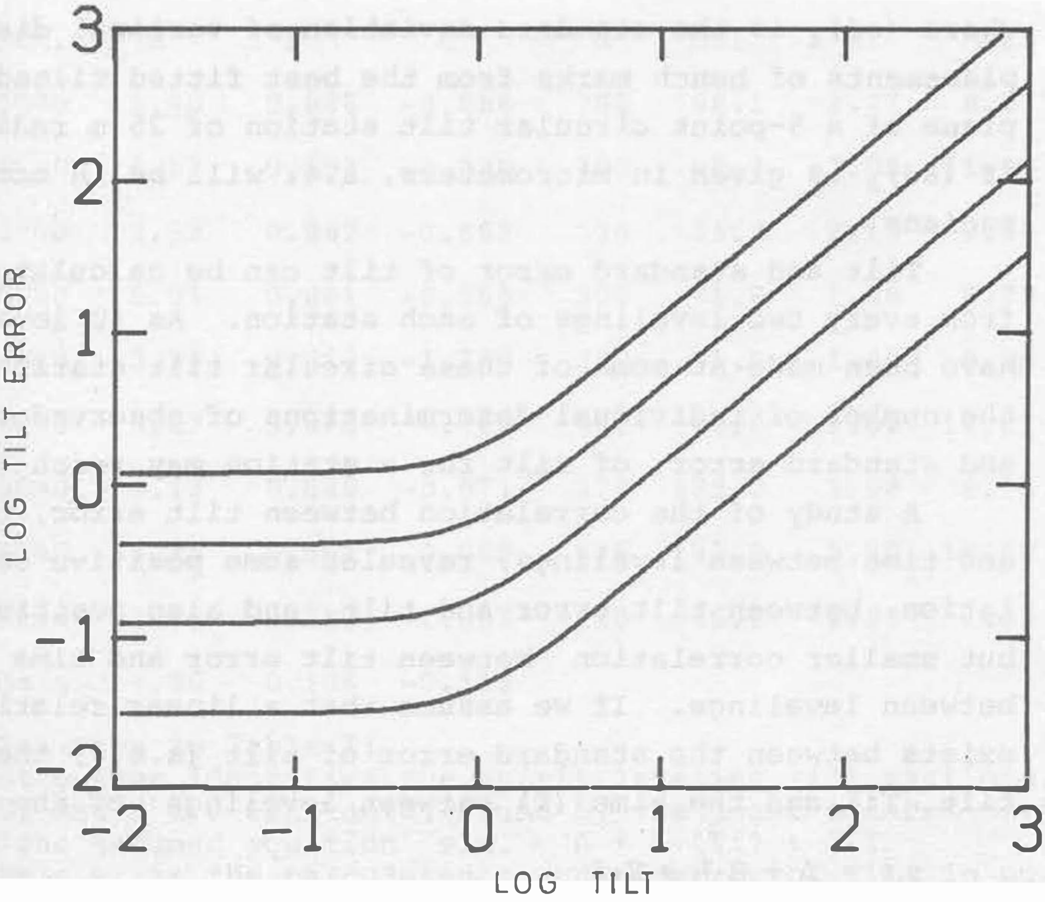

Fig. 2. Calculated standard error of tilt as function of regional tilt from equation 10. Scales are logarithmic and the unit on both scales is microradian. Curves are drawn for linear profiles of 50 m length with 2 marker (top), 200 m length with 5 marker (second from top), 1000 m length with 11 marker (third from top), and 5000 m length with 51 marker (bottom). If regional tilt is less than one microradian, the tilt error is dominated by errors of observations, but for greater regional tilt, the irregular surface deformation (wrincles) dominate the error of tilt.

ANALYSIS OF TILT ERRORS AT CIRCULAR OPTICAL LEVEL TILT STATIONS

We accept the standard error of observed tilt (s.e.) between two levelings as given by equation (3) which says:

$$s.e. = (sd)_z / 39.5 \tag{11}$$

where $(sd)_z$ is the standard deviation of vertical displacements of bench marks from the best fitted tilted plane of a 5-point circular tilt station of 25 m radius. If $(sd)_z$ is given in micrometers, s.e. will be in microradians.

Tilt and standard error of tilt can be calculated from every two levelings of each station. As 40 levelings have been made at some of these circular tilt stations, the number of individual determinations of observed tilt and standard error of tilt for a station may reach 780.

A study of the correlation between tilt error, tilt, and time between levelings, revealed some positive correlation between tilt error and tilt, and also positive, but smaller correlation between tilt error and time between levelings. If we assume that a linear relation exists between the standard error of tilt (s.e.), the tilt (Ti) and the time (T) between levelings of the form:

$$s.e. = A + B \cdot Ti + C \cdot T \tag{12}$$

we can calculate the constants A, B, and C by the least squares method from several hundred data points for each tilt station. Table I shows the value of these constants for all except one of the circular optical leveling tilt stations, which have been observed over extended period before 1982 in the Krafla-Mývatn area. The station 0010 (Leirhnjúkur) is omitted in this study because of extreme deformation associated with fault movements within the area of the tilt station.

We can also assume that the standard error of tilt is a linear function of the time between levelings only, or a linear function of the observed tilt only.

TABLE I.

The apparent combined effect of observed tilt and time between levelings on the standard error of tilt at circular spirit leveling tilt stations in North Iceland

Stn.	A	B	C	N	MTi	MT	Mse
0000	5.40	0.035	-0.086	780	98.1	2.27	8.97
0020	6.57	0.197	-2.030	300	48.0	2.08	11.82
0040	3.62	0.082	-0.662	120	55.3	2.13	6.64
0050	5.01	0.061	-0.565	300	41.9	1.88	6.78
0060	3.38	0.213	-1.760	300	41.4	1.85	8.92
0070	3.57	0.078	4.482	465	40.0	1.81	14.81
0080	5.13	0.039	-0.871	378	122.2	1.98	8.15
0090	6.33	0.051	2.408	435	145.9	1.90	18.39
0200	3.83	0.202	-1.987	210	36.2	1.89	7.38
Mean	4.80	0.106	-0.119				

Explanation to Table I:
First column identifies the spirit leveling tilt stations. A, B, and C are constants, found by the least squares method, for the assumed equation $s.e. = A + B \cdot (Ti) + C \cdot T$ where s.e. is the calculated standard error of tilt in mu. rad.. Ti is the observed tilt in mu. rad., T is the time between observations in years, N is the number of tilt calculations, MTi is the average of all tilt values, MT is the average time in years between observations used in calculating the tilt, Mse is the average of all calculated standard errors of tilt in mu. rad. Tilt, time, and tilt error is calculated for every period between two observations, which includes great amount of overlapping in time.

TABLE II

The apparent effect of time between levelings on the standard error of tilt at circular spirit leveling tilt stations in North Iceland.

Stn.	A(T)	B(T)	R^2	seR
0000	6.53	1.08	0.140	0.031
0020	9.91	1.33	0.055	0.055
0040	4.83	0.86	0.153	0.077
0050	5.85	0.50	0.030	0.056
0060	7.28	0.90	0.035	0.056
0070	5.52	5.15	0.367	0.029
0080	6.26	0.94	0.069	0.048
0090	8.70	5.08	0.456	0.026
0200	5.22	1.15	0.117	0.061
Mean	6.68	1.89	(0.158)	

Explanation to Table II.
First column identifies the spirit leveling tilt stations. A(T) and B(T) are constants found by the least squares method for the assumed linear equation:
$s.e. = A(T) + B(T) \cdot T$, where T is the time in years between observations and s.e. is the calculated standard error of tilt in mu. rad. R is the coefficient of correlation between standard error of tilt and time between observations, and seR is the standard error of R. Number of data points is as given in Table I.

TABLE III

The apparent effect of observed tilt on the standard error of tilt at circular spirit leveling tilt stations in North Iceland.

Stn.	A(Ti)	B(Ti)	R^2	seR
0000	5.73	0.033	0.195	0.032
0020	4.91	0.144	0.417	0.034
0040	3.48	0.057	0.376	0.057
0050	4.86	0.046	0.108	0.052
0060	2.47	0.156	0.423	0.033
0070	8.63	0.154	0.162	0.039
0080	4.90	0.027	0.182	0.042
0090	7.09	0.077	0.507	0.024
0200	3.40	0.110	0.363	0.044
Mean	5.05	0.089	(0.304)	

Explanation to Table III:
First column identifies the spirit leveling tilt stations. A(Ti) and B(Ti) are constants found by the least squares method for the assumed equation:
s.e. = A(Ti) + B(Ti)·Ti, where s.e. is the calculated standard error of tilt and Ti is the calculated tilt in mu. rad. R is the coefficient of correlation between s.e. and Ti, and seR is the standard error of R. Number of data points is as given in Table I.

These linear relations can be written in the following form:

$$\text{s.e.} = A(T) + B(T)\cdot T \tag{13}$$

and

$$\text{s.e.} = A(Ti) + B(Ti)\cdot Ti \tag{14}$$

The calculated values of the constant A(T), B(T), A(Ti), and B(Ti) are given in Tables II and III.

It can be argued, that the assumed linear relation should be replaced by a relation which includes the square root of the sum of two or three squared terms:

$$\text{s.e.} = \sqrt{a^2 + (b\cdot Ti)^2 + (c\cdot T)^2}\ ,\quad \text{s.e.} = \sqrt{(a(T))^2 + (b(T)\cdot T)^2}\ ,\quad \text{s.e.} = \sqrt{(a(Ti))^2 + (b(Ti)\cdot Ti)^2}$$

That procedure is more correct from a statistical point of view, but the great range of observed tilt and the great scatter of calculated standard errors (Fig. 3), makes it dubious. That method will give the value of a(Ti) 50 ± 22 per cent higher than A(Ti) in Table III, and the value of b(Ti) 28 ± 24 per cent higher than B(Ti) in Table III, but the coefficient of correlation would not improve significantly.

The numerical values presented in the three tables show a very definite relation between standard error of tilt and observed tilt, in such a way that the standard error of tilt increases with increased calculated tilt. The rate of increase of tilt error for the best linear fit, can be given as percentage of the observed tilt, which ranges between about 3% at stations 0000 and 0080, and about 15% at the stations 0020, 0060, and 0070. The average value is about 9% (Fig. 3)

The relation between tilt error and time between levelings is much less clear. Table I indicates that tilt error usually decreases with increased time between observations, which is against any expected trend. However, the significant correlation between tilt and time between levelings may cause the value of B and C in Table I to be of no significance.

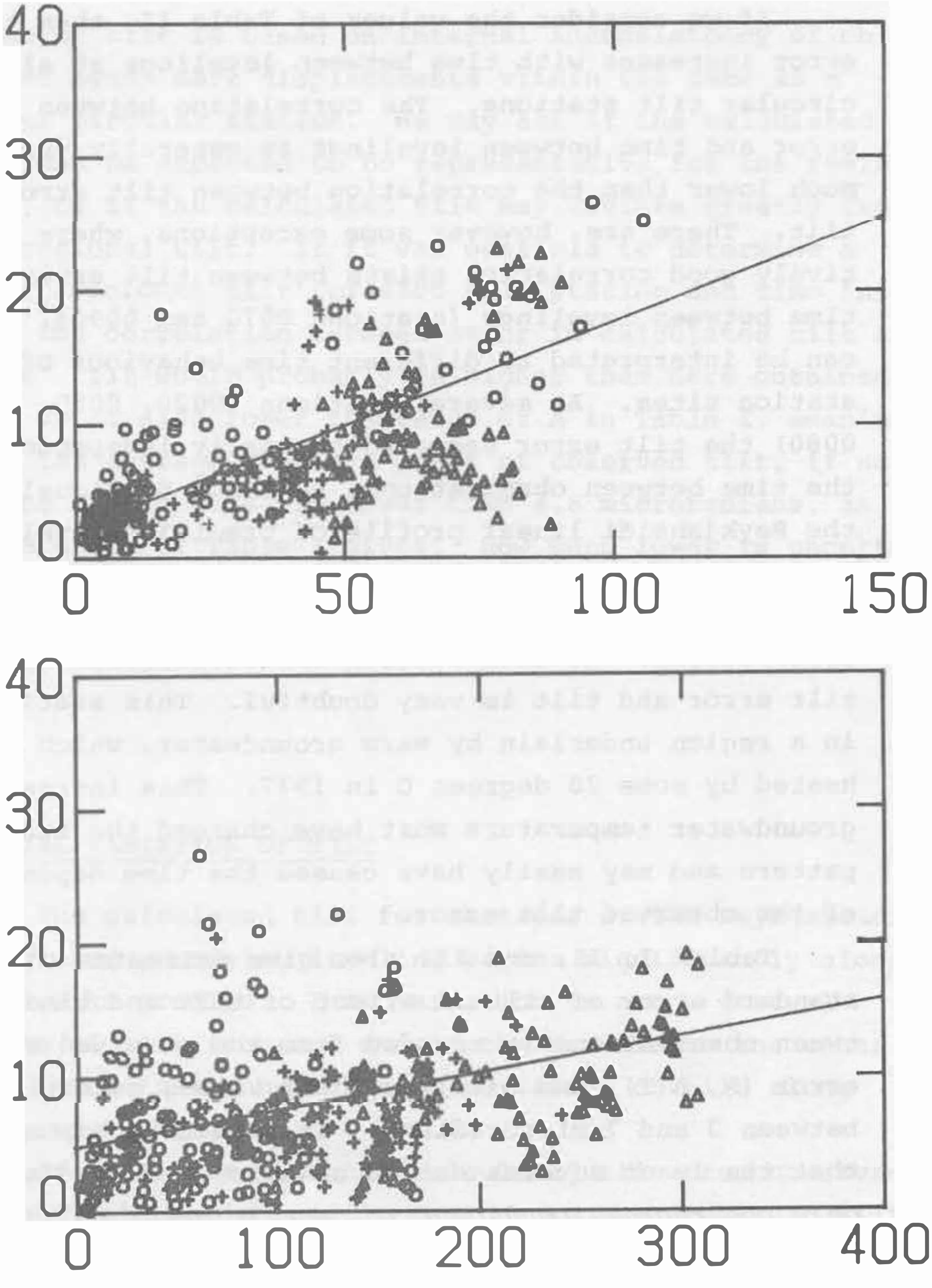

Fig. 3. Calculated standard error of tilt at circular tilt stations versus magnitude of calculated tilt. Tilt and tilt error are calculated for every interval between levelings. Observations are divided into three classes after the length of time between levelings. Open circles represent shortest time between levelings, less than 500 days, intermediate time, 500 to 1000 days is shown by crosses, and longest time, more than 1000 days, is represented by triangles. Shown are all observations at two stations, station 0060 (Grjótagjá-N) with rather large increase of tilt error with tilt (top), and 0080 (Ytri Bjarghóll) with small increase in tilt error with tilt (bottom). Vertical scale is calculated standard error of tilt in microradians and the horizontal scale is observed tilt, also in microradians.

If we consider the values of Table II, then tilt error increases with time between levelings at all the circular tilt stations. The correlation between tilt error and time between levelings is generally very low, much lower than the correlation between tilt error and tilt. There are, however some exceptions, where relatively good correlation exists between tilt error and time between levelings (stations 0070 and 0090). This can be interpreted as different time behaviour of the station sites. At several stations (0020, 0050, 0060, 0080) the tilt error seems to be nearly independent of the time between observations, as found previously for the Reykjaheidi linear profile of precision leveling.

The station 0070 shows the most pronounced increase of standard error of tilt with increased time between observations. At this station the correlation between tilt error and tilt is very doubtful. This station lies in a region underlain by warm groundwater, which was heated by some 20 degrees C in 1977. This increase in groundwater temperature must have changed the stress pattern and may easily have caused the time dependance of the observed tilt error.

Tables I, II, and III also give estimates of the standard error of tilt if effect of tilt and time between observations is removed from the observed standard error (A, A(T), and A(Ti)). These values generally range between 3 and 7 microradians. It should be emphasized, that the least squares method as here used determines only one regression line fitting tilt error to the tilt and/or time, and the great scatter of individual values of the standard error tends to calculate too high values for the constants A, A(T), and A(Ti). The same effect will underestimate the value of B, B(T), and B(Ti), and C. This is especially true for the constants B and B(T), which are calculated from data points of very low correlation.

We have here compared the tilt error with the observed tilt, as calculated from observation of 25 m radius circular tilt station. The calculated standard error of tilt is based on internal inconsistency of observed bench mark displacements within the same 25 m radius circular station. We may ask if the calculated tilt can be expected to be representative for the regional tilt, or if the calculated tilt may deviate greatly from the regional tilt. If it was possible to determine a "true" regional tilt for each tilt station and time interval, the correlation between error in calculated tilt and "true" tilt would probably be higher than here obtained. This would also lower the value of A in Table I, meaning that the average standard error of observed tilt, if no ground tilt occurs, is lower than 4.8 microradians, as the average of Table I gives. How much lower is uncertain, and cannot be resolved by statistical means from the present data.

SPATIAL VARIATION OF TILT

The calculated tilt for sections of the Reykjaheidi profile of precision leveling varies significantly along the 3.3 km profile. This variation is not a gradual change of tilt from one end of the profile to the other end, but rather a near random variation of tilt along the profile (Fig. 4).

If we assume that the calculated tilt from observations of the whole profile represents the "tectonic tilt" of the area, then we can determine the deviation of the observed tilt of sections of the profile from the "tectonic tilt". This has been done for all sections of 4 and 6 adjacent bench marks for all periods between levelings (Table IV). The standard deviation of profile section tilt from the "tectonic tilt" varies with the length of the sections, and also with the "tectonic tilt".

From Table IV we see that the standard deviation of profile section tilt from the "tectonic tilt" is roughly

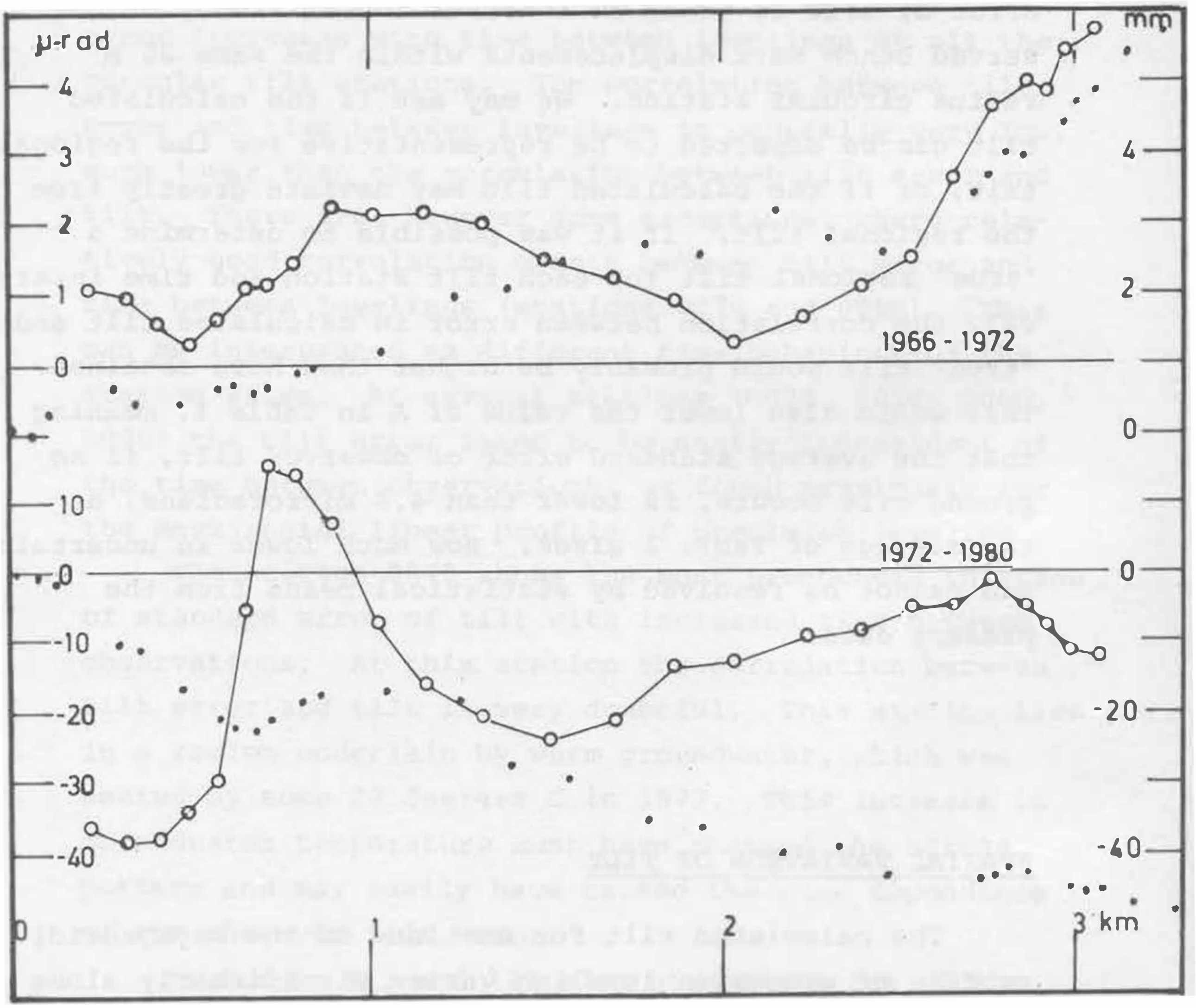

Fig. 4. Observed tilt along the Reykjaheidi linear profile of precision leveling in 1966-1972 (upper curve) and 1972-1980 (lower curve). Open circles connected by thin lines give the tilt component in the general direction of the profile, calculated from vertical displacements of every 6 adjacent bench marks (left scale). Dots show the vertical displacements of each bench mark relative to bench mark 301 at the left edge of the diagram and the east end of the profile (right scale). Positive tilt is up towards west. The first period (1966-1972) was tectonically quiet while the second period was characterized by widespread rifting and volcanic and seismic activity, commencing in 1975. Note the different scales.

Table IV

Standard deviation of calculated tilt of sections of the Reykjaheidi profile of precision leveling, from the average tilt of the whole profile, for several observational periods.

Period	Tect. tilt	Standard dev. from tectonic tilt 4-marker sect. 240 m	 526 m	 6-marker sect. 419 m	 841 m
1966-68	0.849	1.194	0.692	0.756	0.520
1968-70	0.514	0.988	0.483	0.910	0.312
1970-72	0.184	1.365	0.590	1.106	0.523
1966-70	1.363	1.909	0.703	1.520	0.382
1968-72	0.698	1.272	0.338	1.108	0.267
1966-72	1.547	1.953	0.912	1.679	0.682
1972-76	3.006	4.308	4.191	3.918	2.922
1976-80	9.943	19.093	6.660	15.280	4.678
1972-80	12.949	21.94	9.10	18.43	6.43

Explanation to Table IV:
The 4-marker sections of the leveling profile were divided into two groups after their length, and the same was done to the 6-marker profile sections. The average length of the 4-marker sections was 240 m and 526 m respectively in the two groups, and that of the 6-marker sections 419 m and 841 m. The second column gives the average tilt along the whole Reykjaheidi profile of precision leveling as calculated by the least squares method from all 30 bench marks in microradians. Columns 3, 4, 5, and 6 give the standard deviation of profile section tilt from the values of column 2 for the short and long 4-marker sections and the short and long 6-marker sections respectively, also in microradians.

proportional to the "tectonic tilt", and inversely proportional to the length of the profile sections. If this relation is accepted, the standard deviation of profile section tilt from tectonic tilt can be written in the form:

$$(sd)_s = A \cdot Ti / L \qquad (15)$$

and if the profile length is in km, the most probable value of the constant A is 0.450 for the 4-marker profile sections, and 0.617 for the 6-marker profile sections, or an average value of 0.533 if the number of bench marks is not of significance in this estimate.

The probable error of the constant A (above) is quite large, as the individual values calculated from the data of Table IV range from 0.26 to 1.17. The standard deviation of these values from the average is about 0.2, so we can estimate the standard deviation of observed tilt from tectonic tilt as a function of the length of the leveling profile L (in km) and the tilt Ti as:

$$(sd)_s = (0.5 \pm 0.2)\ Ti / L \qquad (16)$$

These deviations are caused by the irregular deformation of the ground as tectonic deformation takes place.

VARIATION OF TILT WITH TIME

The tectonic processes are usually considered as slow and continuous over extended time span. Exceptions are rapid non-elastic processes associated with earthquakes, volcanic eruptions, and creep.

The four levelings of the Reykjaheidi linear profile of precision leveling, prior to the 1975-(1982) volcano-tectonic episode, were made during a period of slow elastic deformation. The observed variation of tilt with time, during that period, can be considered as irregularities of the time response of the earth's surface to a steady tectonic process.

Fig. 5 shows the observed ground tilt along the Reykjaheidi leveling profile for the three 2-year periods between levelings, 1966-1968, 1968-1970, and 1970-1972. The tilt shown is calculated from observed vertical displacements of 6 adjacent bench marks. The observed variation of tilt along the leveling profile varies greatly from one 2-year period to another, and no obvious correlation is seen between the profile section tilt over two 2-year periods.

Fig. 5 shows also the difference in observed tilt at two consecutive 2-year periods, calculated for every section of 6 adjacent bench marks of the profile. This difference varies along the profile in somewhat similar way, as does the observed tilt, and no obvious correlation exists between the two tilt difference curves, D-1, and D-2.

Interpretation of the data presented in Fig. 5 is rather difficult, but it shows that observed variation of tilt along a linear tilt profile cannot be expected to be repeated at a later period of observation. Also, variation of tilt from one 2-year period to another should not be interpreted as an indication of secular trend in the rate of ground tilt.

Fig. 5. Observed tilt component parallel to the Reykjaheidi linear profile of precision leveling for the three 2-year intervals between levelings prior to the 1975-(1982) volcanotectonic episode (top), and difference in observed ground tilt from one 2-year period to another (bottom). Tilt and tilt difference is calculated from every set of 6 adjacent bench marks. D-1 shows observed tilt 1966-1968 minus observed tilt 1968-1970, and D-2 shows the observed tilt 1968-1970 minus that of 1970-1972. East end of the leveling profile is at the left edge of the diagram, and positive tilt is up towards west. Zones of numerous open fissures and small fault scarps in the recent lava cross the profile at 0.3 to 0.9 km and 2.7 to 3.1 km measured from the east (left) end of the profile. These parts of the profile show greatest irregularities in the tilt.

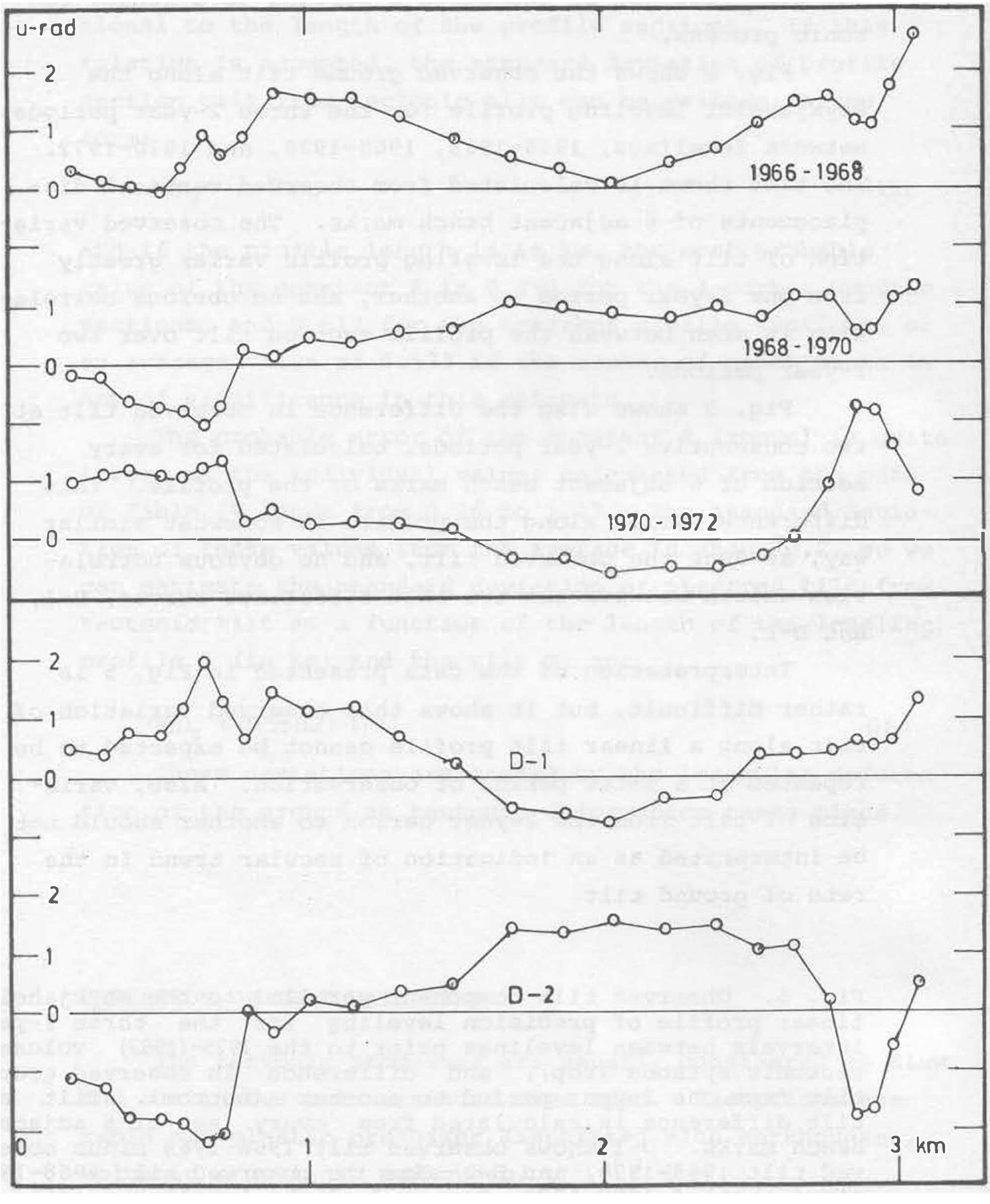

Fig. 5. Caption on page 25.

OBSERVATIONAL TILT ERROR AT THE CIRCULAR OPTICAL LEVELING TILT STATIONS

The observational method applied to the circular optical leveling tilt stations in Iceland include 4 (or 6) readings of the elevation of each bench mark. The scatter of these readings allows us to estimate the accuracy of the determined relative bench mark elevation.

The standard error S(i) of the elevation of bench mark i, relative to the optical level is approximately:

$$S(i) = sd(i)/\sqrt{N-1} \qquad (17)$$

where sd(i) is the standard deviation of individual elevation readings from the mean value, and N is the number of readings, usually 4, of the relative elevation of each marker.

As two levelings are needed for each tilt observation, the average standard error of the relative displacement of each bench mark will be approximately:

$$S = mS(i)\sqrt{2} = msd(i)\sqrt{2/(N-1)} \qquad (18)$$

where mS(i) is the average of the standard error, S(i) of the two observations, and msd(i) is the average of the standard deviation of individual elevation readings from the mean, for the two levelings entering into the tilt calculation.

The average observational standard error of tilt then becomes for 5-marker station of 25 m radius approximately:

$$(s.e.)_0 = mmS/39.5 \qquad (19)$$

where mmS is the average value of S as defined in previous paragraph, taken over all 5 bench marks of the tilt station.

The value of S(i) was calculated for all levelings of 1980 to 1982 at the 7 stations, 0020, 0040, 0050, 0060, 0070, 0080, and 0090. For the 5 first stations, the average value of sd(i) was found to be about 110 micrometers, but about 145 micrometers for the last two stations. This difference is understandable as the stations 0080 and 0090 were observed several times under conditions far from ideal because of large expected tilt.

These observational errors in relative bench mark elevation will cause average standard error of computed tilt $(s.e.)_o$ equal:

$$(s.e.)_o = 2.3 \text{ mu. rad.} \tag{20}$$

at the stations 0020, 0040, 0050, 0060, and 0070, and similar average observational error was found at the stations 0000 and 0200. The average standard observational error of tilt at the stations 0080 and 0090 are found to be about 3.0 microradians, and similar observational error was found for the station 0010.

This average value of the standard error of tilt due to errors of levelings, is in fair agreement with the value of "A" of Tables I, II, and III. The "A" values are known to be somewhat too high for the tilt error reduced to zero tilt and zero time between levelings, because of the methods used in the calculation of the regression line.

The present average value of 2.3 to 3.0 microradians for the average observational standard error of tilt can be accepted as an average value for the tilt observations at the circular optical leveling tilt stations in the Krafla-Mývatn region in 1976 to 1982. If tilt observations are made only when the weather conditions are favorable, this error can be reduced considerable, probably to about 1.5 microradians.

CONCLUSIONS

Repeated optical leveling is a simple tool to observe tectonic ground tilt, but great caution must be taken in interpreting the results.

Deviation of observed tilt from the tectonic tilt is a complicated function of the geology and topography of the area where observations are made, and also of the magnitude of tectonic tilt, in addition to the effect of observational errors.

For a 3.3 km linear profile of repeated precision leveling in North Iceland, the standard deviation of observed tilt from average tectonic tilt was found to be approximately proportional to the magnitude of the tectonic tilt, and inversely proportional to the length of the leveling profile (equation 16).

The proportionality constant is in this case about 0.5, if the length is in km. This means, that for a leveling profile of 0.5 km length, or less, the deviation of observed tilt from the tectonic tilt is frequently as large as or larger than the magnitude of the tectonic tilt.

The standard error of observed tilt, as computed from the deviation of vertical displacements of individual bench marks from the best linear relation between vertical displacements and horizontal distance along the profile, depends on the magnitude of tilt, the length of the leveling profile, and the number of permanent bench marks. For sections of the 3.3 km Reykjaheidi profile of precision leveling, the standard error (s.e.) of tilt component along the general direction of the profile is found to approximate an exponential function of both profile length (L), and number (N) of bench marks, and a linear function of the magnitude of the tectonic tilt. The proportionality constant is here about 0.25, if the profile length is in km, which is significantly less than that found from deviation of profile sections tilt from the tectonic tilt. The exponentional constants are approximately -0.6 with respect to profile length, and -0.27 with respect to number of bench marks. Observational errors also contribute to the error in calculated tilt (equation 10).

We assume that observational error of tilt, and error of tectonic tilt due to the tilt related steam, are entirely unrelated. Then the accumulated error of these two causes will be equal the square root of the sum of the square of the errors caused by each source. If we further assume, that these two error sources have the same dependance on profile length and number of bench marks (which is actually not probable), then an equation giving the most probable standard error of tilt from two levelings will be:

$$s.e. = (s.e.)_0 \cdot N^{-0.27} \cdot L^{-0.6} \qquad (21)$$

where

$$(s.e.)_0 = \sqrt{0.24^2 + (0.26 \cdot Ti)^2} \qquad (22)$$

and s.e. is the most probable standard error of tilt, L is the profile length in km, N is the number of permanent bench marks, Ti is the tectonic tilt (actually the average tilt of the whole profile), and $(s.e.)_0$ is the standard error of observed tilt reduced to unit (1 km) profile length, and "unit" number of bench marks. The unit of the tilt and error of tilt is microradian, and the constant o.24 is the most probable standard error of tilt of a unit length, "unit" marker profile if tectonic tilt is zero.

The standard error of tilt as obtained from 25 m radius circular optical leveling tilt stations in North Iceland, is found to be approximately proportional to the observed tilt. In absence of observable ground tilt, this error is 2.3 to 3.0 microradians on the average for observations in 1980 to 1982, but can be reduced to about 1.5 microradians if only days of favourable weather conditions are used for the optical leveling of the stations. This is somewhat greater error than that calculated from the above equations, which give 0.95 microradians for the most probable standard error of tilt of 50 m profile consisting of 5 bench marks. This difference may lie in different observational technique.

The increase in standard error of tilt, as calculated from deviation of vertical displacements of individual bench marks from plane tilt, with increased observed tilt, appears to range from 3 per cent to 15 per cent of the observed tilt at the various circular optical leveling tilt stations.

There is no conclusive evidence for increase of error in observed ground tilt with increase of time between the two levelings used to calculate the tilt.

The increase in error of observed tilt with increasing tilt can be interpreted as formation of irregular wrincles on the earth's surface as the ground is tilted, and the amplitude of these wrincles is roughly proportional to the ground tilt. The wave length of these wrincles is variable. The observations of the Reykjaheidi linear profile of precision leveling indicate a predominate wave length of 1.5 to 2 km (Fig. 4), while the results of observations of the 25 m radius circular tilt stations indicate significant amplitudes of waves of much shorter wave length (Tryggvason, 1979).

The amplitudes of these wrincles, as observed on the Reykjaheidi linear profile, appear to be up to 10 mm during the 1972-1980 period, when average tilt was 13 microradians, but only about 1 mm in the 1966-1972 period, when the observed tilt was about 1.5 microradians. Short wave length wrincles have less amplitude.

These surface wrincles may be considered as the response of the inhomogeneous surface layer to the horizontal and shear strain, which is associated with the observed ground tilt. As they are controlled by the surface geology, their characteristics cannot be expected to be similar, with respect to amplitude and wave length, in a region of different geology.

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